Supervenience: A Survey

Alex Steinberg

The world we live in is structured. Some aspects of reality depend on further aspects, and some facts completely determine others. In a famous metaphor due to Kripke (1980, 153f.): when God created the world he had to decide on many things. But he did not have to decide on everything explicitly: decisions on some questions already determined answers to others. To take an uncontroversial example: once the truth-functionally atomic facts were settled, all the truth-functionally complex facts were settled as well. God did not need to specify that (grass is green and snow is white), on top of specifying that grass is green and specifying that snow is white. Or consider the specific shades of colour. Once God had decided on their distribution, he did not have to further specify the distribution of red, green and blue.

Many philosophers think that an important sort of dependence that relates conjunctive facts to atomic facts and the primary colours to the specific shades can be captured by supervenience claims. A difference in conjunctive facts requires a difference in atomic facts, and if there is a difference in the primary colours, this difference will be consequential on a difference in the specific shades. Traditionally, slightly more controversial theses were discussed. When R. M. Hare introduced the term ‘supervene’ into the current philosophical debate (in his 1952, 80f.) he was concerned with what he called value-words such as ‘good picture’. According to Hare, there cannot be two things that are otherwise completely alike, while one is a good picture and the other is not. G. E. Moore (1922, 261) is often cited as an early champion of the supervenience of moral on nonmoral properties. And Donald Davidson (1970, 88) endorsed the perhaps most widely discussed supervenience thesis of the mental on the physical.
Supervenience, then, promises to be one of the dependence relations that structure the world we live in. This paper aims to give an overview of the subject. Section 1 introduces the main kinds of supervenience. Section 2 discusses their relations. And section 3 makes the case that purely modal definitions of supervenience can fruitfully be improved upon.

In the rest of the paper I will follow the bulk of the philosophical literature in focusing exclusively on properties as the relata of supervenience. This means that, on the one hand, while some philosophers have maintained supervenience theses between, e.g., facts, expressions or events, they are not in the purview of this paper. Perhaps, not much is lost, since these latter supervenience theses may be reducible to cases of property supervenience. This claim will remain undisputed as well. On the other hand, it means that even relations will be ignored. This may be a graver oversight. Mental relations, for instance, should certainly be covered by the claim that the mental supervenes on the physical. Luckily, the oversight is remedied by Stephan Leuenberger’s contribution to this volume.

1. Varieties of Supervenience

According to David Lewis,

[t]o say that so-and-so supervenes on such-and-such is to say that there can be no difference in respect of so-and-so without difference in respect of such-and-such. (Lewis 1983, 358)

Many agree. McLaughlin and Bennett (2008, §1) call this the ‘core idea’ of supervenience. It will be our point of departure. It will turn out that different notions correspond to the slogan. We start with some preliminaries.

1.1 Preliminaries: Properties, Sets and Worlds

At the end of my introductory remarks I already indicated a constraint on how the ‘so-and-so’ and ‘such-and-such’ in the Lewis quotation is to be replaced. Given our decision to focus on property supervenience, the replacements should be names for single properties

1 For this claim about fact supervenience, see e.g. Kim 1984, 169.
or for pluralities of them. Thus, such claims as the claim that mental properties supervene on physical properties and the claim that the property of being red supervenes on the particular shades of red are what concerns us here.\(^2\) In discussions of supervenience a conception of properties as lightweight and abundant is typically assumed. The properties at issue do not have to be fundamental explainers or whatever else natural, sparse, properties may be good for.\(^3\) Rather, talk of properties in supervenience theses is a device for generalisation and classification. When we ask whether mental properties supervene on physical properties, such questions as the following are pertinent:

1. Can there be two things which are physically exactly alike, while one is in pain and the other is not?
2. If someone believes that snow is white and someone else does not, do they have to be physically dissimilar as well?

Questions of the naturalness of sub- and supervenience candidates, on the other hand, are simply not relevant here.\(^4\)

For present purposes we do not need to decide between various conceptions of properties according to which there are enough of them.\(^5\) For properties to be able to fulfil their role in the pertinent generalisations, it is sufficient that the following inferences turn out to be valid, paradoxes aside:\(^6\)

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\(^2\) For convenience I assume that colours as well as specific shades of colour are properties. Those who doubt the view should replace (shades of) colour talk with talk about the property of having this-or-that (shade of) colour in the examples to follow.

\(^3\) For a range of potential other uses see Lewis 1983.

\(^4\) In fact, it is a plausible but non-trivial view that all other properties supervene on the natural properties. It would be trivial if the natural properties were the only properties there are.

\(^5\) For specific suggestions see e.g. Lewis 1983; Schiffer 2003; Künne 2006.

\(^6\) The instances of (H) and (¬H) in which ‘F’ is replaced by ‘self-exemplifying’ cannot be valid. Otherwise, the property of being self-exemplifying would be self-exemplifying just in case it is not, which is impossible.
(H) \( \frac{a \text{ is } F}{a \text{ has the property of being } F} \) 

(¬H) \( \frac{a \text{ is not } F}{a \text{ does not have the property of being } F} \)

Whichever conception of properties validates (H) and (¬H) ensures that questions like (1) and (2) are directly relevant to our supervenience claims.

When supervenience claims are formulated for pluralities of properties, this is typically done with the help of sets. For instance, let COLOUR be the set of primary colours and SHADE be the set of specific shades of colour. Then the claim that the primary colours supervene on the specific shades would typically be stated as the claim that there cannot be a difference with respect to COLOUR-properties without a difference with respect to SHADE-properties; where, necessarily, something is an \( A \)-property just in case it is a property that is an element of \( A \).7 This should be borne in mind, since set talk and natural readings of the more colloquial formulations may come apart. Suppose for instance that Ben only thinks about the properties of being red, blue and green on 21st September 2012. Let BEN be the set of properties Ben thought about on 21st September 2012 and consider the following two claims:

(3) There cannot be a difference with respect to BEN-properties without a difference with respect to SHADE-properties; and

(4) There cannot be a difference with respect to the properties thought about by Ben on 21st September 2012 without a difference in the specific shades of colour.

Since sets have their members essentially, BEN (aka COLOUR) has the primary colours as its only elements in every world in which it exists. But, of course, in many worlds Ben thinks about other properties on 21st September 2012, e.g. only about the property of being a car. Let

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7 I use upper case letters from the beginning of the alphabet as variables for sets. Perhaps this is a good place to indicate further such conventions for individual variables: ‘\( w \)’, ‘\( u \)’ and ‘\( v \)’ (as well as their primed and subscripted forms) are variables for possible worlds; ‘\( \forall \)’ and ‘\( Q \)’ are variables for properties; ‘\( \phi \)’ and ‘\( \psi \)’ are variables for facts, and ‘\( x \)’, ‘\( y \)’ and ‘\( z \)’ are variables for individuals of any kind.
be such a world. Then, given the supervenience of the primary colours on the specific shades, in \( w \) no two objects will differ in \( \text{Ben} \)-properties without differing in \( \text{Shade} \)-properties, but there may well be a bicycle and a car sporting the very same shade of blue in \( w \).

This difference, however, will typically not show up. In the interesting cases, the supervenience candidates are usually specified in terms of their \textit{necessary properties}: being a mental property, being a physical property, being a primary colour and so forth. While there is a world at which being red is not thought about by Ben on 21st September 2012, there is no world at which it is not a primary colour. Likewise for being blue and being green. Thus, in every world each \( \text{Colour} \)-property will also be a (primary) colour property and vice versa. In such a case set talk and the more colloquial formulation will be in line.\(^8\)

Like our discussion up to this point, the supervenience literature makes frequent use of possible worlds. Since supervenience is characterised in modal terms—there can be no difference in respect of so-and-so without a difference in respect of such-and-such—this is hardly surprising. As in the case of properties, no substantial account of possible worlds needs to be presupposed. There just have to be enough of them for the following inferences to be valid:

\[
(\emptyset) \quad \text{It is possible that } p \quad \frac{\text{There is a possible world at which } p}{(\Box)} \quad \text{It is necessary that } p
\]

It does not matter, for instance, whether possible worlds are taken to be maximal sums of spatio-temporally related individuals (Lewis 1986), maximal possible states of affairs (Plantinga 1974) or most specific properties the universe might have had (Stalnaker 2003). Any straightforward account of possible worlds will aim to validate \( (\emptyset) \) and \( (\Box) \). The reader is free to take her pick. It is an interesting question whether the supervenience debate could in principle do without

\[\begin{align*}
(\Box) & \quad \exists x (F_x \iff \Box F_x) \land \forall x (x \in A \iff F_x) \land \Box \exists x (x = A) \rightarrow \Box \forall x (F_x \iff x \in A).
\end{align*}\]

\(^8\) More generally, this is because the following is a theorem of modal set theory:

\[\begin{align*}
(\Box) & \quad \exists x (F_x \iff \Box F_x) \land \forall x (x \in A \iff F_x) \land \Box \exists x (x = A) \rightarrow \Box \forall x (F_x \iff x \in A).
\end{align*}\]
possible worlds or whether they are indispensable in the characterisation of what supervenience amounts to. We will return to this issue in section 1.3 below.

1.2 Weak and Strong Supervenience

The supervenience slogan that there can be no difference in \(A\)-properties—the supervenient properties—without a difference in \(B\)-properties—the subvenient or base properties—can be spelled out in various ways. At least since Kim 1984 three notions that correspond to different such ways are recognised: the notions of weak, strong and global supervenience. Weak and strong supervenience concern differences in \(A\)- and \(B\)-properties between individuals. They are discussed in this subsection. Global supervenience concerns the distribution of \(A\)- and \(B\)-properties over whole worlds. It is delegated to section 1.4.

The difference between weak and strong supervenience can be brought out by focusing on a modal and a possibilist explication of the supervenience slogan:

- **Modal** There can be no two individuals with different \(A\)-properties but the same \(B\)-properties;
- **Possibilist** There are no two possible individuals with different \(A\)-properties but the same \(B\)-properties.

Consider the set of all birth country properties, COUNTRY, and the set of all birthplace properties, PLACE. A property is an element of COUNTRY just in case it is the property of being born in (some place which on 21st September 2012 belongs to) \(C\), for some country \(C\). Similarly, a property is in PLACE just in case it is the property of being born at geo-coordinates \(G\), for some geo-coordinates \(G\). No two people who agree in PLACE-properties disagree in COUNTRY-

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9 The latter stance is taken, e.g., in Lewis 1986, 14f.
10 The first is called modal because it uses the modal auxiliary ‘can’. The possibilist explication talks about possible, perhaps non-actual, individuals. It is called possibilist, since only the possibilist believes that his quantifiers can reach beyond the actual.
11 The parenthesis is needed in order to deal with changes in country borders. For the sake of brevity I will ignore it in the remainder.
properties. If two people are born at the very same place in central London, perhaps in the same hospital bed a few years apart, they were both born in the UK, if they were both born at some place at the outskirts of Berlin, they were both born in Germany, and so forth.

It is quite plausible that COUNTRY- and PLACE-properties satisfy (Modal). A counterexample would describe a world \( w \) in which two people \( a \) and \( b \) are both born at some specific place \( G \) but in different countries \( C \) and \( C' \) (even though no territorial changes occurred). This would be so, if at \( w \) there were places at which one is located in \( C \) just in case one satisfies some further condition, being left-handed, say, and in \( C' \) otherwise. But, arguably, there is no such world. Country-location can only be a matter of geographical position. If \( C \) and \( C' \) had attempted to ‘share’ a place in the way envisioned they wouldn’t have succeeded. If this generalises, COUNTRY- and PLACE-properties satisfy (Modal): COUNTRY-properties \textit{weakly supervene} on PLACE-properties.

Whether or not they satisfy (Modal), COUNTRY- and PLACE-properties surely do not satisfy (Possibilist). Places that actually belong to one country could have belonged to a different country. For instance, although Berlin actually belongs to Germany, there is a possible world \( w \) at which Berlin belongs to Poland. Now consider \( a \), a possible individual at the actual world \( @ \) that was born at some place at the outskirts of Berlin \( G \), and another possible individual of \( w \), \( b \), which was likewise born at place \( G \). \( a \) has all the same PLACE-properties at \( @ \) as \( b \) has at \( w \)—they both have the property of being born at \( G \) and no others. Still, they differ in their COUNTRY-properties: at \( @ \), \( a \) has the property of being born in Germany and lacks the property of being born in Poland. At \( w \), \( b \) has the property of being born in Poland, but lacks the property of being born in Germany. COUNTRY- and PLACE-properties, thus, do not satisfy (Possibilist): COUNTRY-properties do not \textit{strongly supervene} on PLACE-properties.

To get a unified characterisation of weak and strong supervenience, let us follow the literature in introducing the 5-place predicate ‘\( x \) in \( w \)’
is indiscernible from \( y \) in \( w' \) with respect to \( A \)-properties—or ‘\( x \) in \( w \) is \( A \)-indiscernible from \( y \) in \( w' \)’ for short—as follows:\(^{12}\)

**INDISCERNIBILITY**

\( x \) in \( w \) is indiscernible from \( y \) in \( w' \) with respect to \( A \)-properties \( \iff \)
\( \forall P \in A \ (at \ w, \ x \ has \ P \iff \ at \ w', \ y \ has \ P) \).

If \( x \) in \( w \) is not \( A \)-indiscernible from \( y \) in \( w' \), there is some \( A \)-property over whose possession \( x \) and \( y \) disagree in the respective worlds. In this case we say that \( x \) in \( w \) is \( A \)-discernible from \( y \) in \( w' \).

With this notion of indiscernibility at hand, we can give the following characterisations of weak and strong supervenience:

**Weak**  \( A \)-properties weakly supervene on \( B \)-properties \( \iff \)
\( \forall w, x, y \ (x \ in \ w \ is \ B \)-indiscernible from \( y \) in \( w \) \( \rightarrow \) \( x \) in \( w \) is \( A \)-indiscernible from \( y \) in \( w \))

**Strong** \( A \)-properties strongly supervene on \( B \)-properties \( \iff \)
\( \forall w, w', x, y \ (x \ in \ w \ is \ B \)-indiscernible from \( y \) in \( w' \) \( \rightarrow \) \( x \) in \( w \) is \( A \)-indiscernible from \( y \) in \( w' \))

We have already encountered a plausible example of properties which weakly but not strongly supervene on other properties: COUNTRY- and PLACE-properties. If \( x \) in \( w \) has all the same PLACE-properties as \( y \) in \( w' \), it is plausible that \( x \) in \( w \) is also COUNTRY-indiscernible from \( y \) in \( w' \), as long as \( w = w' \). COLOUR- and SHADE-properties, on the other hand provide a plausible example of strong supervenience: if \( x \) in \( w \) is SHADE-indiscernible from \( y \) in \( w' \), \( x \) in \( w \) will have all the same COLOUR-properties as \( y \) in \( w' \) as well, whether or not \( w = w' \). COLOUR-properties also weakly supervene on SHADE-properties. In fact, since the case in which \( w = w' \) is a special case of the definition of strong supervenience, it is always the case that \( A \)-properties weakly

\(^{12}\) It is slightly unfortunate that the word ‘indiscernible’ is used here, since it tends to have epistemic overtones. As the definition makes clear, they are not intended. A related definition defines \( A \)-indiscernibility between individual-world pairs in a similar way. On the assumption that possible individuals exist at exactly one world and have properties only there (or simpliciter, not relative to a world), a 3-place relation simply between possible individuals could have been used instead. Nothing of substance hinges on this. The discussion in the main text is easily adjustable.
Supervene on B-properties, provided they strongly supervene. Further entailment relations between kinds of supervenience are discussed in section 2.

Sometimes only weak or strong supervenience theses of less-than-full strength may be plausible. Consider for instance the thesis that mental properties supervene on physical properties. One may want to subscribe to such a view only in a qualified way. For, one may think that there are possible worlds very unlike our own in which mental properties float free of physical properties. These worlds, one may want to hold, are just not *physically* possible relative to the actual world. Full-strength supervenience will be falsified by them nevertheless. What is consistent with the existence of such far-out dualist worlds are supervenience theses whose world-quantifiers are restricted to physically possible worlds. In explicit form, such restricted supervenience theses look as follows:

**Weak**<sub>R</sub> \( \forall w (w \in F \rightarrow \forall x, y (x \in w \text{ is } B\text{-indiscernible from } y \text{ in } w \rightarrow x \in w \text{ is } A\text{-indiscernible from } y \text{ in } w)) \).

**Strong**<sub>R</sub> \( \forall w, w' ((w \in F \land w' \in G) \rightarrow \forall x, y (x \in w \text{ is } B\text{-indiscernible from } y \text{ in } w' \rightarrow x \in w \text{ is } A\text{-indiscernible from } y \text{ in } w')) \).

Here ‘\( F \)’ and ‘\( G \)’ are place-holders for predicates expressing restrictions on the domains of the world quantifiers. These restrictions may be quite involved. For instance, the domain of the second world-quantifier in (Strong<sub>R</sub>) may be a function of the domain of the first. Strong supervenience of MENTAL-properties on PHYSICAL-properties may be a case in point. In line with the discussion above, we might want to replace ‘\( F \)’ with ‘world physically possible with respect to \( @ \)’ and ‘\( G \)’ with ‘world physically possible with respect to \( w' \)’. This would ensure that only worlds that are physically possible with respect to worlds that are physically possible from the actual world are considered. However, for the remainder of this survey we will ignore such restrictions and turn back to full-strength supervenience theses.
1.3 Alternative Formulations

Jaegwon Kim (1984, 163) argues that when the supervenience base \( B \) is closed under Boolean operations the notion defined by (Weak) above is equivalent to the one characterised as follows which we will call \( \text{weak supervenience}_K \) (‘\( \Box \)’ is the box of modal logic, to be read as ‘it is necessary that’):

\[ \forall x; P ((P \in A \land x \text{ has } P) \rightarrow \exists P'(P' \in B \land x \text{ has } P' \land \forall y (y \text{ has } P' \rightarrow y \text{ has } P))). \]

Let us say that a property \( P' \) is materially sufficient for a property \( P \) just in case everything with \( P' \) also has \( P \). Then (Weak\(_K\)) can be put in words thus: \( A \)-properties weakly supervene\(_K\) on \( B \)-properties just in case, necessarily, everything with an \( A \)-property also has some \( B \)-property that is materially sufficient for the \( A \)-property.

Kim (1987, 317f.) also showed that there is a similar characterisation of \( \text{strong supervenience}_K \):

\[ \forall x; P ((P \in A \land x \text{ has } P) \rightarrow \exists P'(P' \in B \land x \text{ has } P' \land \Box \forall y (y \text{ has } P' \rightarrow y \text{ has } P))). \]

Let us say that \( P' \) is strictly sufficient for \( P \) just in case it is necessary that everything with \( P' \) also has \( P \). Then (Strong\(_K\)) says that \( A \)-properties strongly supervene\(_K\) on \( B \)-properties just in case, necessarily, everything with an \( A \)-property also has a \( B \)-property which is strictly sufficient for the \( A \)-property.

The initial characterisation of strong supervenience differs from that of weak supervenience in containing an additional world quantifier. In this way, not only \emph{intra}-world but also \emph{inter}-world indiscernibility with respect to \( A \)- and \( B \)-properties is relevant for strong supervenience. In a similar fashion, the difference between Kim’s alternative characterisation of weak and strong supervenience is merely one in the strength of the connection between \( A \)- and \( B \)-properties. While weak supervenience requires that for every \( A \)-property there is a \textit{materially} sufficient \( B \)-property in every world, strong supervenience requires that for every \( A \)-property there is a \textit{strictly} sufficient such \( B \)-property.
As an illustration of how the alternative characterisations work reconsider COUNTRY- and PLACE-properties. It is plausible that, necessarily, for every COUNTRY-property \( P \) there is some PLACE-property \( P' \) such that everything with \( P' \) also has \( P \). As it happens, there are many such PLACE-properties for each COUNTRY-property. Every place in Berlin is such that being born there is materially sufficient for being born in Germany, for instance. In general: necessarily, for each country \( C \) and geo-coordinates \( G \), being born at \( G \) is materially sufficient for the property of being born in \( C \), provided that \( G \) belongs to \( C \). But even though many PLACE-properties are materially sufficient for each COUNTRY-property, none of them is strictly sufficient. For instance, no place in Berlin is such that the property of being born there is strictly sufficient for the property of being born in Germany, since area now covered by Berlin could have belonged to Poland instead of Germany, for instance. In general: no place in any country is such that being born at the former is strictly sufficient for being born in the latter—places do not essentially belong to countries.

Thus, COUNTRY-properties weakly but not strongly supervene on PLACE-properties according to both characterisations. Let us now confirm that they always give the same verdict as long as the base set is closed under Boolean operations.

A set of properties \( A \) is said to be closed under Boolean operations just in case whenever \( P \) is in \( A \) and \( B \) is a subset of \( A \), some \( Q \) and \( Q' \) that satisfy the following conditions are in \( A \) as well:

\[
\text{Neg} \quad \Box \forall x \Box (x \text{ has } Q \leftrightarrow \neg x \text{ has } P) \\
\text{Con} \quad \Box \forall x \Box (x \text{ has } Q' \leftrightarrow \forall P (P \in B \rightarrow x \text{ has } P')).
\]

If \( P \) and \( Q \) satisfy (Neg), we may call \( Q \) a negation of \( P \). If \( \{P_1, \ldots, P_n\} \) and \( Q' \) satisfy (Con), we may say that \( Q' \) is a conjunction of \( P_1 \) and \( \ldots \) and \( P_n \). This indicates why talk of Boolean closure is appropriate.

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13 If \( B \) is a subset of \( A \) \( (B \subseteq A) \), every element of \( B \) is also an element of \( A \). If every element of \( A \) is an element of \( B \) we say that \( B \) is a superset of \( A \) \( (B \supseteq A) \). And the principle of extensionality ensures us that if \( B \) is both a subset and a superset of \( A \), then \( B = A \).

14 Since \( B \) may be infinite if \( A \) is, satisfaction of (Con) may require ‘infinite’ conjunctive properties to be in \( A \).
Whether there is a *unique* negation of a given property, and whether there is a *unique* conjunction of a given set of properties depends on questions of property individuation. On some accounts, properties are individuated intensionally. That is, according to these accounts, there are no two properties that are necessarily co-exemplified: if $P$ is strictly sufficient for $P'$ and $P'$ is strictly sufficient for $P$, then $P = P'$. On such a conception, any property will have at most one negation, and for any $P$ and $P'$ there will be at most one conjunction. For definiteness, consider a Lewisian conception of properties as sets of possible individuals. Any such property will have exactly one negation, its complement with respect to the set of all possible individuals, and all sets of properties $B$ will have exactly one conjunction, their intersection. For simplicity but inessentially, I assume in what follows that negation and conjunction are unique.

The following is a proof to the effect that if $B$ is closed under Boolean operations, $A$-properties strongly supervene on $B$-properties just in case the former strongly supervene$_K$ on the latter.15 To avoid duplication I focus on strong supervenience in the remainder of this section. (The proof of the same result for weak supervenience and weak supervenience$_K$ is a straightforward variant.)

**RIGHT TO LEFT:** Suppose that $A$-properties do not strongly supervene on $B$-properties. Then there are some worlds $w$ and $w'$ and individuals $x$ and $y$ such that $x$ in $w$ is $A$-discernible but $B$-indiscernible from $y$ in $w'$. Consequently, there is some $A$-property $P$ which is such that either (i) $x$ has $P$ at $w$ and $y$ does not have $P$ at $w$ or (ii) $x$ does not have $P$ at $w$ and $y$ has $P$ at $w'$. If (i) is the case, at $w$, $x$ has an $A$-property, $P$, but no $B$-property that is strictly sufficient for $P$, since at $w'$, $y$ has the $B$-properties $x$ has at $w$ but not $P$. If (ii) is the case, at $w'$, $y$ has $P$ but no $B$-property strictly sufficient for $P$. Either way, it is possible that there is an individual with an $A$-property but without any $B$-property strictly sufficient for that $A$-property. If $A$-properties do not strongly supervene on $B$-properties, $A$-properties do not strongly supervene$_K$ on $B$-properties.

**LEFT TO RIGHT:** Suppose that (i) $A$-properties strongly supervene on $B$-properties and that (ii) $x$ has some $A$-property $Q$ at some world $w$.

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15 Cp. e.g. Kim 1987, 317f.
Let $B_{x,w}$ be the set of $B$-properties $x$ has at $w$. So, by (Con), there is a $P \in B$, the conjunction of all $B$-properties $x$ has at $w$, such that

$$\square \forall \chi \square (\chi \text{ has } P \leftrightarrow \forall P'(P' \in B_{x,w} \rightarrow \chi \text{ has } P>').$$

Call that property $\cap B_{x,w}$—the maximal $B$-property $x$ has at $w$. Trivially, $x$ has $\cap B_{x,w}$ at $w$. It is easy to see that $\cap B_{x,w}$ is strictly sufficient for $Q$. Consider any $y$ with $\cap B_{x,w}$ at any $w'$. Since $y$ has $\cap B_{x,w}$ at $w'$, (iii) $x$ in $w$ is $B$-indiscernible from $y$ in $w'$. By (i) and (iii), $x$ in $w$ is $A$-indiscernible from $y$ in $w'$. So, by (ii), $y$ has $Q$ at $w'$. Since $x$ and $w$ were chosen arbitrarily and $Q$ was an arbitrary $A$-property, $A$-properties strongly supervene on $B$-properties, provided the former supervene strongly on the latter.

To see that (iii) holds, consider any $B$-property $P$. Either (a) $x$ has $P$ at $w$ or (b) $x$ does not have $P$ at $w$. If (a), $P \in B_{x,w}$. Hence,

$$\square \forall \chi \square (\chi \text{ has } \cap B_{x,w} \rightarrow \chi \text{ has } P).$$

So, $y$ has $P$ at $w$. If (b), by (Neg), there is a $P' \in B$, the negation of $P$, such that

$$\square \forall \chi \square (\chi \text{ has } P' \leftrightarrow \neg \chi \text{ has } P).$$

Since, $x$ does not have $P$ at $w$, $x$ has $P'$ at $w$, and $P' \in \cap B_{x,w}$. So, by the same reasoning as above, $y$ has $P'$ at $w'$. Since $P'$ is the negation of $P$, $y$ does not have $P$ at $w'$. Since $P$ was an arbitrary $B$-property, $x$ in $w$ is $B$-indiscernible from $y$ in $w'$.

The proof is beyond dispute. It is often pointed out, however, that the result is too weak to license the claim that strong supervenience and strong supervenience are equivalent notions. It is easy to see that they are not. For, consider a set of properties, NECESSARY, that contains only necessary properties—properties had by everything at every world—and a set of properties, IMPOSSIBLE, that contains only impossible properties—properties not had by anything at any world. NECESSARY-properties strongly supervene on IMPOSSIBLE-properties. For, any $x$ at any $w$ is indiscernible from any $y$ at any $w'$ with respect to NECESSARY-properties full stop. So, a fortiori, they are indiscernible in these respects, provided they are indiscernible with respect to IMPOSSIBLE-properties as well. However, NECESSARY-
properties do not strongly supervene\textsubscript{K} on \textsc{Impossible}-properties, since my coffee mug has a \textsc{necessary}-property but no \textsc{Impossible}-property strictly sufficient for it, since it has no \textsc{Impossible}-property \textit{full stop}.

This result is not surprising, since the above proof relies on the assumption that \textsc{a}- and \textsc{b}-properties are closed under Boolean operations, while \textsc{impossible} (and \textsc{necessary}) are not. In fact, the Boolean closure of \textsc{impossible}—the smallest superset of \textsc{impossible} closed under Boolean operations—simply is the Boolean closure of \textsc{necessary}, so that ‘they’ trivially strongly supervise\textsubscript{K}. But many sets we are particularly interested in may not be closed under Boolean operations either. This may be either so because (i) not every property has a negation and not every set of properties has a conjunction, or (ii) even though they do, negations of \textsc{a}-properties and conjunctions of \textsc{a}-properties need not be \textsc{a}-properties themselves.\textsuperscript{16}

The first worry should perhaps not impress us too much. As discussed in section 1.1 above, the conception of properties typically employed in the supervenience debate is an abundant one. On such a conception it is hard to see how the existence of negations or conjunctions can be denied. In fact, on Lewis’s account, for instance, we have already seen that it is easily provable that (Neg) and (Con) are always satisfiable. The second worry cannot be so easily dismissed, however. For, suppose that negations of \textsc{a}-properties are always \textsc{a}-properties. Then numbers would have mental properties, for instance, simply because they do not think. Similarly, thoughts would have weight properties, simply because they do not weigh 5 kg. These examples make it likely that virtually no set of properties we are typically interested in will be closed under Boolean operations. But when the sets are not closed under Boolean operations, the two notions may give different verdicts. In fact, since the proof of the right to left direction does not appeal to Boolean closure, strong supervenience\textsubscript{K} has been shown to be strictly stronger than strong supervenience: whenever \textsc{a}-properties strongly supervise\textsubscript{K} on \textsc{b}-properties, they strongly supervise, \textit{but not vice versa}.

\textsuperscript{16} See, e.g., McLaughlin 1995.
We should not overrate this result, however.\textsuperscript{17} It does not show, for instance, that in characterising some forms of supervenience—weak and strong supervenience as originally defined—we cannot do without appeal to possible worlds. Nor does it mean that Kim has simply added to the family of supervenience notions his own pair of stricter-than-usual ones. For, in the immediate vicinity of supervenience there is a characterisation of a notion of strong supervenience that is provably equivalent to our original one.

In fact, there are two. An obvious variant of \((\text{Strong}_K)\) replaces ‘\(B\)’ in the definiens throughout with ‘\(\mathbf{B}(B)\)’ which designates the Boolean closure of \(B\). The Boolean closure of a set of properties \(B\) is the smallest set which contains all properties in \(B\) and is closed under \((\text{Neg})\) and \((\text{Con})\). It is easy to see that this characterisation agrees with the original one, whether or not \(B\) itself is closed under Boolean operations. The crucial fact here is that \(A\)-properties strongly supervene on \(B\)-properties just in case \(A\)-properties strongly supervene on \(\mathbf{B}(B)\)-properties.

The second variant is discussed and exploited by Fabrice Correia in his 2005, chapter 6. Since it will play a role in further developments in section 3, it deserves some more attention. Correia’s variant utilises the fact that we can speak of properties in the plural, saying that they are jointly had by an individual. The idea is that—unlike the definiens of \((\text{Strong}_K)\)—we should not require that for every \(A\)-property there is some single \(B\)-property strictly sufficient for the \(A\)-property. Rather, we should only require that for every \(A\)-property there are some \(B\)-properties which are collectively sufficient for the \(A\)-property.

Let us use double letters as plural variables, making the necessary grammatical changes in the sentences that contain them. \(\exists QQ (QQ \in B \land x \text{ has } QQ)\), for instance, is to be read as ‘There are some properties such that they are elements of \(B\) and \(x\) has them’, where this is understood to be true already when \(x\) has only one \(B\)-property but may be made true by several \(B\)-properties collectively. Let us further use expressions like ‘\(x\) has \(QQ\)’ to mean ‘\(QQ\) are all the \(B\)-properties \(x\) has’. Now consider the following variant of \((\text{Strong}_K)\):

\textsuperscript{17} As some of the literature, e.g., McLaughlin 1995, tends to do.
Let us say that $B$-properties $QQ$ are jointly strictly sufficient for property $P$ just in case it is necessary that whenever $QQ$ are all the $B$-properties something has, that thing also has $P$. Then we can paraphrase $(\text{Strong}_C)$ as follows: $A$-properties supervene on $B$-properties just in case it is necessary that whenever something has an $A$-property either (i) its $B$-properties are jointly strictly sufficient for the $A$-property or else (ii) it has no $B$-properties, while having no $B$-properties at all is strictly sufficient for the $A$-property. To see that the notion defined by $(\text{Strong}_C)$ is equivalent with the original supervenience notion, recall the crucial left-to-right direction of the original proof. There, $\cap B_{x,w}$ was used in order to show that, assuming that $B$-properties are closed under conjunction, there is some single $B$-property strictly sufficient for the $A$-property. Instead we can now employ $\cap B_{x,w}$ directly. For, either $\cap B_{x,w}$ has elements or it does not. If it has elements then its elements make true the existential quantification of the first disjunct of $(\text{Strong}_C)$’s definiens. If it has no elements, the second disjunct of $(\text{Strong}_C)$’s definiens will be true. Either way, if $A$-properties strongly supervene on $B$-properties, the former strongly supervene on the latter, whether or not $B$-properties are closed under Boolean operations. Since this is so, we can drop the subscript and think of strong supervenience in terms of $(\text{Strong}_C)$’s definiens. Strictly analogous considerations apply to weak supervenience, of course.

1.4 Global Supervenience

In addition to the local notions of weak and strong supervenience philosophers have been interested in global supervenience. Sometimes, it is plausible to think, the world-wide distribution of one kind of properties $B$ fixes the world-wide distribution of another kind of properties $A$, even when the individual-by-individual distribution of

\[ 18 \text{ Pace Correia (2005, 137) the second disjunct is needed.} \]
B-properties does not so fix the distribution of A-properties. Consider for instance the set of properties \( \text{WEIGHT} \) and the property \( \text{MOSTWEIGHT} \). A property is an element of \( \text{WEIGHT} \) just in case it is the property of weighing \( n \) kg, for some number \( n \). \( \text{MOSTWEIGHT} \) is the property of weighing at least as much as anything else. Now, intuitively, worlds that agree in the distribution of \( \text{WEIGHT} \)-properties agree in their distribution of \( \text{MOSTWEIGHT} \). If both at \( w_1 \) and \( w_2 \) Fred has the property of weighing 80 kg and nothing has the property of weighing \( 80+m \) kg for some positive number \( m \), then Fred has the property of weighing at least as much as anything else at both worlds. This generalises: \( \text{MOSTWEIGHT} \) globally supervenes on \( \text{WEIGHT} \)-properties. However, \( \text{MOSTWEIGHT} \) does not strongly supervene on \( \text{WEIGHT} \)-properties.\(^{19}\) We can see this by considering a further world \( w_3 \) which is like \( w_2 \) apart from containing an extra thing, Frank, that weighs 81 kg. At \( w_1 \), Fred weighs at least as much as anything else, but Fred has no \( \text{WEIGHT} \)-properties that are strictly sufficient for having \( \text{MOSTWEIGHT} \). For, at \( w_3 \), Fred weighs 80 kg as well, but does not weigh at least as much as anything else, since Frank weighs more than Fred.

A more controversial but (philosophical) real life example in which global but not individual supervenience may be present is the mental and the physical. Suppose mental content is partly determined externally. Perhaps, e.g., at \( w_1 \), Fred on earth thinks about water when physically indiscernible Frank on far-away twin-earth thinks about twater.\(^{20}\) Then \( \text{MENTAL} \)-properties do not even weakly supervene on \( \text{PHYSICAL} \)-properties, since Fred and Frank have the same \( \text{PHYSICAL} \)-properties at \( w_1 \) but different \( \text{MENTAL} \)-properties (Fred thinks about water, Frank does not). Still, for all that has been said it may be that \( \text{MENTAL} \)-properties globally supervene on \( \text{PHYSICAL} \)-properties, since all worlds that agree on their distribution of \( \text{PHYSICAL} \)-

\(^{19}\) It is customary in the supervenience debate to speak of the supervenience of some property \( P \), when only a single property rather than a plurality of them is at issue. Strictly speaking, such claims should be understood to concern \( P \)'s \textit{unit set}, \( \{P\} \).

\(^{20}\) For this to be plausible Fred and Frank have to be unlike any Freddies and Franks we are likely to encounter at the actual world: they cannot themselves consist to a large part of water. The twin earth scenario is due to Putnam (1973).
properties will agree on which thinkers are able to have water as opposed to twater thoughts.

In our informal discussion we have worked with the idea that \(A\)-properties globally supervene on \(B\)-properties just in case all worlds that agree on their distribution of \(B\)-properties also agree on their distribution of \(A\)-properties. It is time to make this idea precise. When do worlds \(w\) and \(w'\) agree on their distribution of \(A\)-properties? As, e.g., Paull and Sider (1992, 834) have pointed out, the most straightforward understanding—that \(w\) and \(w'\) themselves should have the same \(A\)-properties—is not pertinent here. For, typically, we are interested in supervenience theses concerning properties that are not had by possible worlds at all. Since possible worlds do not have mental properties, for instance, no worlds disagree (in this sense) on their distribution of mental properties, and, thus, mental properties would vacuously supervene on properties of any kind.

When Kim (1984, §4) introduced the notion of global supervenience he suggested that instead of considering whether the relevant worlds have certain \(A\)-properties we should rather consider which individuals have those \(A\)-properties at them. More specifically, \(w\) and \(w'\) agree in their distribution of \(A\)-properties, according to Kim (they agree, for short), just in case for every possible individual \(x\) and \(A\)-property \(P\), either \(x\) has \(P\) at both \(w\) and \(w'\) or at neither. Plugging this into our initial characterisation, we get the Kimian notion of global supervenience:

\[
\text{GLOBAL SUPERVENIENCE}_K
\]

\(A\)-properties globally supervene\(_K\) on \(B\)-properties \(\iff\) \(df.\)
\[
\forall w, w' \quad (\forall P \in B \forall x (x \text{ has } P \text{ at } w \iff x \text{ has } P \text{ at } w')) \rightarrow \\
\forall P \in A \forall x (x \text{ has } P \text{ at } w \iff x \text{ has } P \text{ at } w')).
\]
Our intuitive discussion conformed to this understanding of what it is for worlds to agree on their distribution of some properties. When we considered whether MOSTWEIGHT supervenes on WEIGHT-properties or whether MENTAL-properties supervene on PHYSICAL-properties, we considered whether there are worlds at which the very same things have the same WEIGHT/PHYSICAL-properties but differ in MOSTWEIGHT/MENTAL-properties. However, more recent discussions of global supervenience favour a more lenient understanding of the notion in question, upon which various alternative notions of global supervenience can be based.

To understand what is at issue consider two worlds $w_4$ and $w_5$. At $w_4$, there are only two individuals $a$ and $b$, $a$ has $P$ and does not have $Q$, while $b$ has $Q$ and lacks $P$. At $w_5$, there are some other individuals $c$ and $d$, $c$ has $P$ and lacks $Q$, while $d$ lacks both $P$ and $Q$ (see figure 1). Do $w_4$ and $w_5$ agree on their distribution of $P$? Well, they do not agree: at $w_4$, $a$ has $P$, while at $w_5$, $a$ does not have $P$. In another sense, however, they do agree. For, at both $w_4$ and $w_5$, there is exactly one thing with $P$ and exactly one thing without $P$. To be precise, each thing that exists in one world can be paired up with exactly one $P$-indiscernible thing at the other.

More formally, let us define the notion of an $A$-isomorphism between world-domains as follows:\footnote{For the sake of brevity, I will often drop mention of the domain in this context and speak about isomorphisms (and bijections generally) between worlds.}

$$
\begin{array}{c}
\begin{array}{c}
w_4 \\
\hline
a \text{ has } P \\
\neg b \text{ has } P \\
\neg a \text{ has } Q \\
b \text{ has } Q \\
\end{array} \\
w_5 \\
\hline
\begin{array}{c}
c \text{ has } P \\
\neg d \text{ has } P \\
\neg c \text{ has } Q \\
\neg d \text{ has } Q \\
\end{array}
\end{array}
$$

Figure 1: $w_4$ and $w_5$
**A-ISOMORPHISM**

$f$ is an $A$-isomorphism between the domains of $w$ and $w' \leftrightarrow_{\delta}$. 

(i) $f$ is a function that maps each element in the domain of $w$ to something in the domain of $w'$ ($f$ is a function from $D(w)$ to $D(w')$) \land

(ii) each element of $D(w')$ is such that $f$ maps something to it ($f$ is onto) \land

(iii) each element of $D(w')$ is such that $f$ maps no more than one thing to it ($f$ is one-one) \land

(iv) $\forall x \in D(w) \forall F \in A$ (at $w$, $x$ has $P \leftrightarrow$ at $w'$, $f(x)$ has $P$).

If conditions (i), (ii) and (iii) are fulfilled, $f$ is a bijection from the domain of $w$ to the domain of $w'$, i.e. $f$ pairs up each thing that exist at one world with exactly one thing that exists at the other. Condition (iv) ensures that the pairs have the same $A$-properties at their respective worlds. For this reason, we may also call $A$-isomorphisms $A$-preserving bijections.

On the present proposal, worlds $w$ and $w'$ agree in their distribution of $A$-properties just in case there is an $A$-isomorphism between them. The identity function on $D(w)$—the function that maps everything that exists at $w$ to itself and does nothing else—is a candidate to be such an $A$-isomorphism. So, whenever $w$ and $w'$ agree in their distribution of $A$-properties, they agree in the present sense. But, as $w_4$ and $w_5$ show, some worlds agree in their distribution of $A$-properties merely in the present sense, since, although their domains are not the same, there is a function that maps them to each other in the right way. In our example, this is the function $f_1$ that maps $a$ to $c$ and $b$ to $d$ (and does nothing else). This is so, since (i) $f_1$ maps each member of $\{a, b\}$ to some element in $\{c, d\}$, (ii) both $c$ and $d$ are the value of $f$ for some argument, namely $a$ and $b$ respectively (iii) neither $c$ nor $d$ are the value of $f_1$ for more than one argument, and (iv) $P$ is exemplified by both $a$ at $w_4$ and $c$, at $w_5$, and lacked by both $b$ at $w_4$ and $d$ at $w_5$. The less mathematically minded reader may picture the inhabitants of the two worlds divided by a boundary but connected across the boundary by pieces of string. The $f_1$-way of tying the string binds

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22 This proposal can be traced back at least to Kim (1988, 115f.) and Paull and Sider (1992, 851).
a to \( c \) and \( b \) to \( d \). In general, \( w \) and \( w' \) will agree in their distribution of \( A \)-properties just in case there is a way of tying the string so that each thing on either side is at the end of exactly one piece of string, no piece of string is left dangling, and all things connected by a piece of string share their \( A \)-properties.

Plugging this more lenient understanding into our initial characterisation we get so-called weak global supervenience:

**Weak Global Supervenience**

\( A \)-properties weakly globally supervene on \( B \)-properties \( \leftrightarrow \) for all \( w, w' \) there exists a \( f \) (is an \( A \)-isomorphism between \( w \) and \( w' \)) such that \( \exists g \) (\( g \) is a \( B \)-isomorphism between \( w \) and \( w' \)).

In words: \( A \)-properties weakly globally supervene on \( B \)-properties just in case all \( A \)-isomorphic worlds are also \( B \)-isomorphic. In our example, for instance, \( P \) does not supervene on \( Q \), since—as already discussed—there is a \( P \)- but no \( Q \)-preserving bijection between \( w_4 \) and \( w_5 \). \( f_1 \) itself is not \( Q \)-preserving, since it pairs \( b \) with \( d \), and \( b \) has \( Q \) at \( w_4 \) while \( d \) lacks \( Q \) at \( w_5 \). The only other bijection between the two worlds, the function \( f_2 \) which pairs \( a \) with \( d \) and \( b \) with \( c \), is not \( Q \)-preserving either, since \( c \) lacks \( Q \) at \( w_5 \). In short: \( w_4 \) has one \( Q \)-thing, while \( w_5 \) has none, so no bijection will be \( Q \)-preserving: \( P \) does not weakly globally supervene on \( Q \).

As the name suggests, weak global supervenience is rather weak. Its weakness consists in the fact that it requires of worlds between which there is an \( A \)-preserving bijection only that there also is some or other \( B \)-preserving bijection. It does not require that any \( A \)-preserving bijection is itself \( B \)-preserving. Consider \( w_4 \) and a world \( w_6 \) which is just like \( w_4 \) except that \( e \) has \( Q \) (see figure 2). \( w_4 \) and \( w_6 \) do not constitute a counterexample to the weak global supervenience of \( P \) on \( Q \), since there is a \( Q \)-isomorphism between \( w_4 \) and \( w_6 \), namely \( f_2 \). However, no bijection between \( w_4 \) and \( w_6 \) is both \( A \)- and \( B \)-preserving. In fact, the distribution of \( Q \) does not seem to be determined by the distribution of \( P \) in any particularly strong sense, since, as \( w_4 \) and \( w_6 \) show, the same distribution of \( P \) allows contrary distributions of \( Q \), i.e. distributions in which the \( P \)-thing plays the \( Q \)-role of a non-\( P \)-thing. As Leuenberger (2009, 117) points out, weak global supervenience of \( A \)-
on $B$-properties merely ensures that the distribution of $B$-properties fixes how many things have $A$-properties.

For applications in which a little less lenience is required, there are stricter notions of global supervenience that require some or even total coordination between $A$- and $B$-preserving bijections. These notions are intermediate and strong global supervenience:

**INTERMEDIATE GLOBAL SUPERVENIENCE**

$A$-properties intermediately globally supervene on $B$-properties $\iff$

$\forall w,w' \ (\exists f \ (f \text{ is an } A\text{-isomorphism between } w \text{ and } w') \implies \exists g \ (g \text{ is an } A\text{-isomorphism between } w \text{ and } w')$.

**STRONG GLOBAL SUPERVENIENCE**

$A$-properties strongly globally supervene on $B$-properties $\iff$

$\exists f \ (f \text{ is an } A\text{-isomorphism between } w \text{ and } w') \implies \forall g \ (g \text{ is an } A\text{-isomorphism between } w \text{ and } w' \implies g \text{ is a } B\text{-isomorphism between } w \text{ and } w')$.

In words: $A$-properties intermediately globally supervene on $B$-properties just in case whenever there is an $A$-isomorphism between worlds, some $A$-isomorphism between them is also a $B$-isomorphism.

---

23 Something like weak global supervenience (albeit with a restriction on admissible bijections) was already suggested in Paull and Sider (1992). Stalnaker (1996, 227) and McLaughlin (1997, 214) seem to be the earliest authors to have distinguished from it the notion of strong global supervenience. The notion of intermediate (or middling, as Bennett calls it) global supervenience was independently introduced by Shagrir (2002) and Bennett (2004).
$A$-properties strongly globally supervene on $B$-properties just in case whenever there is an $A$-isomorphism between worlds, all $A$-isomorphisms between them are also $B$-isomorphisms. Clearly, $w_4$ and $w_6$ show that $P$ neither intermediately nor strongly globally supervenes on $Q$, since $f_i$ is the only $P$-isomorphism between them, while it is not also a $Q$-isomorphism.

It is an interesting question which variant of global supervenience philosophers have in mind when they speak about global supervenience without further specification. Bennett (2004) conjectures that the notion is closest to intermediate global supervenience, since the latter shares the former’s commonly recognised strengths and weaknesses. Leuenberger (2009) argues that it is none of the above, since all of the explications have features that global supervenience is not meant to have. In any case, the notions are sufficiently different for it to seem advisable to keep in mind that there are many relations that may be meant by the term ‘global supervenience’.

Let us end this overview of the different forms of supervenience with a quick look at multiple domain supervenience.

### 1.5 Multiple Domain Supervenience

So far we have been concerned with kinds of supervenience in whose paradigm examples either (i) the distribution of one kind of properties over an individual determines the distribution of another kind (weak and strong (individual) supervenience), or (ii) the distribution of one kind of properties over a whole world determines the distribution of another kind (global supervenience). Multiple domain supervenience caters for needs in between these two extremes. Consider, for instance, wholes and their parts (e.g. a bicycle and its frame, tires and so forth) or constituters and constitutees (like, perhaps, lumps of clay and statues, or bodies and persons). It’s a prima facie plausible view that many if not all properties of wholes supervene on the properties of their parts. Likewise, if statues are not identical with lumps of clay, then, perhaps, at least many if not all properties of the

\[ \forall x (Fx \rightarrow Gx) \]

---

24 The ‘$\exists f$ ($f$ is an $A$-isomorphism between $w$ and $w'$) $\rightarrow$’ part of Strong Global Supervenience is redundant, since ‘$\exists x (Fx \rightarrow Gx)$’ and ‘$\forall x (Fx \rightarrow Gx)$’ are logically equivalent. It is only included for uniformity.
former supervene on properties of the latter. To give just one example, weight properties of wholes seem to supervene on the weight properties of their proper parts. Roughly, no composites have different weights although the distribution of weight properties among their proper parts is the same. In his 1988, Jaegwon Kim introduced multiple domain supervenience in order to provide the resources to frame such theses.

Kim’s basic idea is as follows. While in the case of individual supervenience the very same things have $A$- and $B$-properties, we may be interested in $A$-properties of one kind of things and ask ourselves whether they are correlated with $B$-properties of things that stand in a certain relation to the things that have the $A$-properties, e.g. those that constitute the things with the $A$-properties or those that compose them. The relation in which the bearers of $A$-properties and the bearers of $B$-properties stand is called the coordination relation. The title multiple domain supervenience derives from the typical case in which the coordination relation relates different kinds of things, i.e. different domains. As in the case of individual supervenience, there is a weak and a strong form of multiple domain supervenience, depending on whether only intra- or also inter-world constraints are imposed:\footnote{Cp. Kim 1988, 124f. Unlike the present definition proposals, Kim does not deal with the potential world-relativity of the relation $R$ there.}

**Weak Multiple Domain Supervenience**

$A$-properties weakly supervene$_{MD}$ on $B$-properties with respect to $R \leftrightarrow_{df.}$

\[
\forall w, x, y, C, D \ ( (\forall \zeta (\zeta \in C \leftrightarrow \text{in } w, \zeta \text{ stands in } R \text{ to } x) \land \\
\forall \zeta (\zeta \in D \leftrightarrow \text{in } w, \zeta \text{ stands in } R \text{ to } y) \land \\
C \text{ in } w \text{ is } B\text{-indiscernible from } D \text{ in } w) \rightarrow \\
x \text{ in } w \text{ is } A\text{-indiscernible from } y \text{ in } w).
\]

**Strong Multiple Domain Supervenience**

$A$-properties strongly supervene$_{MD}$ on $B$-properties wrt $R \leftrightarrow_{df.}$

\[
\forall w, w', x, y, C, D \ ( (\forall \zeta (\zeta \in C \leftrightarrow \text{in } w, \zeta \text{ stands in } R \text{ to } x) \land \\
\forall \zeta (\zeta \in D \leftrightarrow \text{in } w', \zeta \text{ stands in } R \text{ to } y) \land \\
C \text{ in } w \text{ is } B\text{-indiscernible from } D \text{ in } w') \rightarrow \\
x \text{ in } w \text{ is } A\text{-indiscernible from } y \text{ in } w').
\]
Although *Weak Multiple Domain Supervenience* and *Strong Multiple Domain Supervenience* look familiar, we do not yet understand their definition properly. For, in both the notion of \( B \)-indiscernibility is applied to sets instead of individuals. What does it mean for a set \( C \) to be indiscernible with respect to \( B \)-properties from a set \( D \)? Kim (1988, 125) tentatively opts for an explication in terms of the existence of a \( B \)-isomorphism between \( C \) and \( D \), a strategy we are already familiar with from our discussion of global supervenience (on pages 111ff. above). Thus, for instance, WEIGHT-properties weakly supervene\(_{MD} \) on themselves with respect to the atomic parthood relation just in case, necessarily, whenever there is a WEIGHT-isomorphism between the atomic parts of \( x \) and \( y \), \( x \) and \( y \) share their WEIGHT-properties as well.

Note that individual supervenience is a special case of multiple domain supervenience as defined here, namely that in which the coordination relation is identity (cp. Kim 1988, 127). Nevertheless, it may be thought that multiple domain supervenience has nothing new to offer. For, whenever there is a case of supervenience\(_{MD} \) of \( A \)- on \( B \)-properties with respect to \( R \), there is a corresponding case of supervenience of \( A \)-properties on properties intimately related to \( B \)-properties, namely on properties of the form *being \( R \)-related to things with this particular distribution of \( B \)-properties* (cf. Kim 1988, 124). However, for reasons of space, these issues cannot be investigated further here. Ralf Bader’s paper in this volume provides a much more detailed discussion of multiple domain supervenience. Let us now turn to the question of what the most important logical relations between forms of individual and global supervenience are.

### 2. Entailment Relations

In the last section we introduced some of the different forms of supervenience that surface in the literature. Since there are so many different notions—David Lewis speaks in an often quoted passage of ‘an unlovely proliferation of non-equivalent definitions’ (Lewis 1986, 14)—it is natural to wonder about their relationship. In particular, a large part of the discussion about supervenience centres around the question of whether supervenience of a certain kind *entails* supervenience of another kind. What is typically at issue here are *universal*
claims, e.g. whether it is always the case that some set of properties $A$ weakly supervenes on a set of properties $B$, given that $A$ strongly supervenes on $B$\textsuperscript{26}. This is how the entailment questions will be understood in what follows.

Some of these questions have rather obvious answers that can be given on the basis of logic alone. As a matter of basic predicate logic, for instance, if everything that is $B$-indiscernible is also $A$-indiscernible, then, a fortiori, all world-mates that are $B$-indiscernible are also $A$-indiscernible. Thus, strong supervenience entails weak supervenience.

Or consider the varieties of global supervenience. Suppose that all $B$-isomorphisms between worlds are $A$-isomorphisms whenever there is an $A$-isomorphism at all. Then, as a matter of basic predicate logic, some $B$-isomorphism between worlds is an $A$-isomorphism whenever there is an $A$-isomorphism. And if the latter is the case, then there is a $B$-isomorphism between worlds whenever there is an $A$-isomorphism. Consequently, strong global supervenience entails intermediate global supervenience, which in turn entails weak global supervenience.

Finally, strong (individual) entails strong global supervenience. If everything that is $B$-indiscernible is $A$-indiscernible, then every bijection between world-domains that only pairs $B$-indiscernible objects only pairs $A$-indiscernible objects.

On the other hand, some entailments obviously do not hold. For instance, we have already seen potential cases of weak without strong supervenience: unsurprisingly, that $B$-indiscernible world-mates are $A$-indiscernible does not guarantee that all $B$-indiscernible possible objects are also $A$-indiscernible. Interestingly, though, as Bacon (1986) points out, if $A$ and $B$ satisfy a further condition (being closed under world-diagonalisation as he calls it), the entailment holds. I provide a simplified variant of Bacon’s result: if $A$- and $B$-properties are closed under world-relativisation and $A$-properties weakly supervene on $B$-properties, $A$-properties strongly supervene on $B$-properties. Closure under world-relativisation ensures that if $w$ is a possible world and $P$

\textsuperscript{26} Talk of entailment is legitimate if a bit grand, since supervenience itself is a modal notion, so that the universal claims in question will be necessarily true if true at all.
is in \( A \), some property \( Q \) is in \( A \) as well that satisfies the following condition:

\[
\text{WR} \quad \Box \forall x \Box (x \text{ has } Q \leftrightarrow \text{ at } w; x \text{ has } P).
\]

Informally, we may think of a given such property as the property of having \( P \) at \( w \) (as in the case of negation and conjunction, legitimacy of the definite article depends on questions of property individuation). The argument is rather straightforward: suppose that \( A \)-properties weakly supervene on \( B \)-properties and that \( A \) and \( B \) are closed under world-relativisation. Let \( c \) and \( d \) be possible individuals and \( w_1 \) and \( w_2 \) possible worlds such that \( c \) at \( w_1 \) is \( A \)-indiscernible from \( d \) at \( w_2 \). Since \( A \) is closed under world-relativisation, for each property \( P \in A \), there is a property \( Q \in A \) such that \( \Box \forall x \Box (x \text{ has } Q \leftrightarrow \text{ at } w_1; x \text{ has } P) \). Call a property that corresponds to \( P \) in this way \( P_{w_1} \). By definition, for any \( P \in A \), \( c \) has \( P \) at \( w_1 \) iff \( c \) has \( P_{w_1} \) at \( w_1 \). By \( A \)-indiscernibility and the fact that \( P_{w_1} \in A \), \( d \) has \( P_{w_1} \) at \( w_2 \) iff \( c \) has \( P_{w_1} \) at \( w_1 \). Again by definition of \( P_{w_1} \), \( d \) has \( P_{w_1} \) at \( w_2 \) iff \( d \) has \( P \) at \( w_1 \). Putting these biconditionals together: for all \( P \in A \), \( c \) has \( P \) at \( w_1 \) iff \( d \) has \( P \) at \( w_1 \). In other words: \( c \) at \( w_1 \) is \( A \)-indiscernible from \( d \) at \( w_1 \). By weak supervenience \( c \) at \( w_1 \) is \( B \)-indiscernible from \( d \) at \( w_1 \). By strictly analogous reasoning, \( c \) at \( w_1 \) is \( B \)-indiscernible from \( d \) at \( w_2 \). Thus, \( A \)-properties strongly supervene on \( B \)-properties.

The remaining entailment questions concerning forms of individual and global supervenience are less straightforward. The history of the question of whether global supervenience entails strong individual supervenience is rather turbulent. Since it has contributed to a better understanding of what is at issue and how (not) to argue for supervenience claims, the main waypoints of the debate will be sketched in what follows.
2.1 The Entailment Claim

Kim’s 1984, 168, contains an attempted proof to the effect that *Global Supervenience* (see p. 112 above) entails strong individual supervenience. The attempted proof, however, is flawed. This was shown by a counterexample due to Bradford Petrie: Suppose logical space is as pictured in figure 3—there are exactly two possible objects *a* and *b* and two possible worlds *w*₁ and *w*₂. At *w*₁, both *a* and *b* have *Q*, while *a* does and *b* does not have *P*. At *w*₂, neither *a* nor *b* have *P*, while *a* does and *b* does not have *Q*. In this case *P* globally supervenes on *Q*, but *P* does not strongly supervene on *Q*. For, *a* at *w*₁ is *P*-discernible but *Q*-indiscernible from *a* at *w*₂. Thus, strong supervenience fails. But, since *b* has *Q* at *w*₁ while *b* does not have *Q* at *w*₂, *w*₁ and *w*₂ do not agree in their distribution of *Q*. A fortiori, they do not agree in their distribution of *P*. Since *w*₁ and *w*₂ are the only two worlds, *P* globally supervenes on *Q*.

Kim (1987, §2) retracts the Entailment Claim, referring to the counterexample just presented. However, Paull and Sider (1992) challenge the pertinence of Petrie’s example to the most pressing question in

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27 Kim merely shows that the fact that *A*-properties do not strongly supervise on *B*-properties does not entail that the former globally supervise on the latter—the second ‘not’ is in the wrong place. The interested reader might want to see for herself. The point at which things go wrong is ‘since *B* does not entail *F*, we can consistently suppose that -*F(χ)* in w#’ (Kim 1984, 168).

28 See Petrie 1987, 121. Petrie’s article was most influential. The claim that Kim’s alleged proof must be mistaken predates Petrie’s paper, however. Petrie himself mentions Hellman 1985. See also Bacon 1986, n7.
this respect: whether global supervenience *metaphysically entails* strong supervenience, i.e. whether it is *metaphysically possible* for there to be sets $A$ and $B$ such that $A$-properties globally supervene on $B$-properties without strongly supervening. What Petrie’s example shows is merely that there is a (strictly) logical possibility, but not all logical possibilities are metaphysically possible: if Kripke (1980, 112ff.) is right, for instance, it is logically possible but metaphysically impossible that Elizabeth II. should be born from any other than her actual parents. More pertinently, it is logically possible but metaphysically impossible that there are *just two* possible worlds.$^{29}$

Indeed, Paull and Sider (1992, 837ff.) argue that, given that there are worlds $w_1$ and $w_2$ as described by Petrie and both $P$ and $Q$ are intrinsic, a plausible *reduplication principle* guarantees that there will be further worlds that falsify global supervenience of $P$ on $Q$. Reduplication principles for possible worlds say that if there are worlds of a certain kind, then there are worlds of some other kind as well, corresponding to intuitive judgements about what is possible given that some other things are. One very simple such recombination principle is what Paull and Sider call the principle of *Isolation:*

**Isolation**

For any world $w$ and individual $x$, if $x$ exists at $w$, then there is a world $w'$ and an individual $y$ such that

(i) only $y$ ($y$'s parts and objects whose existence is necessitated by the existence of $y$ or $y$'s parts) exists at $w'$; and

(ii) $y$ at $w'$ is a duplicate of $x$ at $w$.

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$^{29}$ This is a somewhat charitable reconstruction of the debate. In their paper, Paull and Sider claim that Petrie must have intended to answer the question of metaphysical possibility, albeit using inadequate means (p. 836ff.). However, given that Petrie’s target, Kim (1984), aimed to show that the Entailment Claim holds drawing only on logical resources and closure conditions of the base set, Paull and Sider’s claim about Petrie’s intentions appears doubtful.
Intuitively, *Isolation* says that whenever we have an individual existing at a world, there might have been an isolated duplicate of that thing.\(^{30}\) Now, according to Paull and Sider, ‘duplicates are exactly qualitatively similar considered “as they are in themselves” and not in relation to other things’. Think, for instance, of the Mona Lisa and a perfect facsimile, Fona. Mona and Fona share a great many properties: they have the same size and weight as well as the same colour distribution, for instance. But they may also differ in certain respects: clearly, only one of them is Mona, Fona is worth considerably less than Fona and only Mona was painted by Leonardo da Vinci. Let us follow Paull and Sider (1992, 838) in assuming that duplicates can never differ in intrinsic properties.\(^{31}\)

Given *Isolation* and the intrinsincness of \(P\) and \(Q\), if \(w_1\) exists, then there is also a world \(w_3\) in which only an individual \(c_1\) exists such that \(c_1\) at \(w_3\) is \(P\)- and \(Q\)-indiscernible from \(a\) at \(w_1\). Likewise, if \(w_2\) exists there is also a world \(w_4\) at which only an individual \(c_2\) exists and \(c_2\) at \(w_4\) is \(P\)- and \(Q\)-indiscernible from \(a\) at \(w_2\). That is, if there are worlds like Petrie’s \(w_1\) and \(w_2\), there also are the worlds pictured in figure 4. But these worlds show that \(P\) does not weakly globally supervene on \(Q\).\(^{32}\)

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\(^{30}\) Or rather: a duplicate that is isolated as much as possible. This is meant to be covered by the parenthesis in (i). I will simply ignore difficulties concerning composite objects and ontological dependence in the main text.

\(^{31}\) For our purposes, we may take this as a (partial) elucidation of the notion of an intrinsic property. For more on the intrinsic/extrinsic divide and its relation to duplication see, e.g., Weatherson 2008.

\(^{32}\) Since \(P\) does not weakly globally supervene on \(Q\), \(P\) does not intermediate or strongly globally supervene on \(Q\) either. *Isolation* as it stands does not count against global supervenience\(^k\)—which is quite unfortunate, since this is what the debate was about before Paull and Sider entered the scene. For
For, since exactly one individual exists at \( w_3 \) and \( w_4 \)—\( c_1 \) and \( c_2 \) respectively—there is exactly one bijection from the domain of \( w_3 \) onto the domain of \( w_4 \): the function \( f \) that maps \( c_1 \) to \( c_2 \) and does nothing more. Now, \( f \) is \( Q \)-preserving, since \( c_1 \) has \( Q \) at \( w_3 \) just like \( f(c_1)=c_2 \) at \( w_4 \). However, \( f \) is not \( P \)-preserving, since \( c_1 \) has \( P \) at \( w_3 \) while \( c_2 \) does not have \( P \) at \( w_4 \). Consequently, there are worlds between which there is a \( Q \)-isomorphism but no \( P \)-isomorphism: \( P \) does not weakly globally supervene on \( Q \).

This result generalises: if \( A \) and \( B \) include only intrinsic properties and \( A \)-properties weakly globally supervene on \( B \)-properties, \( A \)-properties strongly supervene on \( B \)-properties. A restricted version of the Entailment Claim is, thus, borne out by a plausible principle about the structure of logical space.\(^{33}\)

It is imperative, however, that the principle is restricted to intrinsic properties, for, as Paull and Sider (1992, §3) also show, it is easy to give an example of properties that strongly individually but not strongly globally supervene on another. Let \( P \) and \( Q \) be any modally

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this, a stronger cousin of *Isolation* would be needed that says that an object existing at a larger world may itself exist in isolation:

**Isolation***

For any world \( w \) and individual \( x \), if \( x \) exists at \( w \) then there is a world \( w' \) such that (i) only \( x \) (\( x \)'s parts and objects whose existence is necessitated by the existence of \( x \) or \( x \)'s parts) exists at \( w' \), and (ii) \( x \) at \( w' \) is a duplicate of \( x \) at \( w \).

Since the shift from *Global Supervenience* \(_K_\) to the more flexible notions of global supervenience described in section 1.4 above occurred around the time of Paull and Sider’s paper, no one seems to have minded that, strictly speaking, they were changing the terms of the debate.

\(^{33}\) Using the resources introduced in the main text, Bennett (2004: Appendix 1) argues for a less restricted entailment claim: if \( A \) includes only intrinsic properties, \( B \) includes only qualitative properties and \( A \)-properties weakly globally supervene on \( B \)-properties, \( A \)-properties strongly supervene on \( B \)-properties. Very roughly, *qualitative* properties are properties that are definable without referring to some individual. All intrinsic properties are supposed to be qualitative, but many extrinsic ones are as well, for instance *being within 1m of a poodle*. As Moyer (2008, n26) points out, however, Bennett’s argument fails because the restriction to qualitative properties is not strong enough to ensure \( B \)-indiscernibility of the isolated duplicates.
independent intrinsic properties, being red and being square, for instance. Let R be any property that is such that, necessarily, a possible object has R just in case it has P and something has Q, i.e.

$$R \equiv \Box \forall x (x \text{ has } R \leftrightarrow (x \text{ has } P \land \exists y \text{ has } Q)).$$

Now, R strongly globally supervenes on \{P, Q\}, i.e. every \{P, Q\}-isomorphism between worlds between which there is any such isomorphism is also an R-isomorphism. For, suppose f is a \{P, Q\}-isomorphism between some worlds \(w_5\) and \(w_6\).

(i) \(\forall x \in D(w_5) (x \text{ has } P \leftrightarrow \text{ at } w_6, f(x) \text{ has } P)\)

and

(ii) \(\forall y \in D(w_5) (y \text{ has } Q \leftrightarrow \text{ at } w_6, f(y) \text{ has } Q)\)

Suppose \(a\) has Q at \(w_5\). Then, by the definition of \(R\), whatever has P at \(w_5\) has R at \(w_5\). By (ii), \(f(a)\) has Q at \(w_5\). Then, again by the definition of \(R\), whatever has P at \(w_6\) has R at \(w_6\). Thus, by (i), \(f\) is R-preserving.

Suppose, on the other hand, nothing has Q at \(w_5\). Then, by (ii), nothing has Q at \(w_6\). But then, by the definition of \(R\), nothing has R at \(w_5\) and nothing has R at \(w_6\). Thus, \(f\) is R-preserving on this supposition as well.

But R does not strongly supervene on \{P, Q\}. For, let \(w_7\) be a world at which \(c\) has P while something other than \(c\) has Q and let \(w_8\) be a world at which \(d\) has P but nothing has Q. Then \(c\) at \(w_7\) is \{P, Q\}-indiscernible from \(d\) at \(w_8\). However, \(c\) at \(w_7\) is R-discernible from \(d\) at \(w_8\), since, by the definition of \(R\), at \(w_7\), \(c\) has R and at \(w_8\), \(d\) does not have R. Consequently, \{P, Q\} and R are a counterexample to the unrestricted Entailment Claim—Strong Global Supervenience does not entail strong individual supervenience, and, therefore, neither does Intermediate Global Supervenience nor Weak Global Supervenience.

2.2 Extending the Base

In the last section we saw that when \(A\) and \(B\) include only intrinsic properties, we have strong supervenience as soon as we have weak global supervenience. On the other hand, when \(A\) and \(B\) include extrinsic properties, we may even have strong global supervenience of \(A\) on \(B\) without strong supervenience of \(A\) on \(B\). Stalnaker (1996,
Appendix) shows, however, that whenever $A$ strongly globally supervenes on $B$, $A$ strongly supervenes on an extension of $B$, $B^+$, where $B^+$ is closed under Boolean operations and maximal-$B$-structural properties (more on this shortly). This section reproduces the proof for Stalnaker’s thesis. It is the last result about logical relations of different supervenience relations discussed in this survey. Note that the corresponding theses concerning weak and intermediate global supervenience do not hold (cf. Bennett 2004, Appendix 3).

The intuitive idea behind Stalnaker’s proof is rather straightforward. Let $\varphi_{B,w}$ be a complete structural description of the instantiation of $B$-properties at a world $w$. That is, let $\varphi_{B,w}$ be the existential closure of a very long existentially quantified conjunction of open sentences that has the following conjuncts:

1. for every individual $x$ that exists at $w$ and every $B$-property $P$: either an open sentence $t_x^P$ has $t_P$ if $x$ has $P$ at $w$ or an open sentence $\neg t_x^P$ has $t_P^\neg$ if $x$ does not have $P$ at $w$; where $t_x^P$ is a variable uniquely associated in the conjunction with $x$ and $t_P^\neg$ is a name of $P$;
2. for every distinct individuals $x$ and $y$ that exist at $w$: $t_x^y \neq t_y^y$.
3. a there-are-no-more-individuals-than-these clause.

Intuitively, $\varphi_{B,w}$ specifies which maximal $B$-properties are instantiated by how many individuals at $w$.\(^{34}\) Let $\varphi_{B,w,x}$ be the property that a possible individual $y$ has at some possible world $w'$ just in case $\varphi_{B,w}$ is satisfied at $w'$ and $y$ is $B$-indiscernible at $w'$ from $x$ at $w$.\(^{35}\) We may call such properties maximal $B$-structural properties. Now let $B^+$ be a superset of $B$ that contains such maximal $B$-structural properties for each world $w$ and individual $x$ that exists at $w$. It can then be shown that whenever $A$-properties do not strongly supervene on the extended set of properties $B^+$, $A$-properties do not strongly globally supervene on $B$-properties.

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\(^{34}\) A maximal $B$-property is a conjunctive property that includes for every $B$-property either it or its negation as a conjunct. Recall p. 107 above.

\(^{35}\) Deleting the initial quantifier that corresponds to $x$ in $\varphi_{B,w}$ results in an open sentence that signifies $\varphi_{B,w,x}$. 
Before we do so, let us specify a closure condition that ensures that $B^+$ contains the required properties. Let $\sigma$ be a sequence of properties in $B^+$ of any (possibly infinite) length. Then $B^+$ also includes a property $P$ such that

\[
SD \quad \square \forall x \quad \square (x \text{ has } P \leftrightarrow \exists \tau \ (\tau \text{ is a sequence in which every object occurs exactly once } \land \forall a \text{ the } a^{th} \text{ item of } \tau \text{ has the } a^{th} \text{ item of } \sigma)) .^{36}
\]

Intuitively, closure under (SD) ensures that there are properties in $B^+$ that a possible individual has at a world just in case there is something in the domain of that world with the first property in $\sigma$, there is something else in the domain of the world with the second property in $\sigma$ ..., and there are no other individuals in the domain. Since $B^+$ is closed under Boolean operations, $B^+$ includes all maximal $B$-properties. Therefore, closure under (SD) and Boolean operations ensures that $B^+$ includes a property a possible individual has at a world just in case, at $w$, there are $n$ individuals with $B$-maximal property $P_1$, another $m$ individuals with $B$-maximal property $P_2$ ..., for any $n, m$ ... and all $B$-maximal properties. Among these properties will be properties corresponding to the complete structural descriptions of the instantiation of $B$-properties at worlds.\textsuperscript{37} Since $B^+$ is closed under conjunction, there will also be properties that an individual has at a world just in case it has one of the properties just mentioned \textit{and} it has one of the $B$-maximal properties. In other words, $B^+$ includes all maximal $B$-structural properties if it is closed under Boolean operations and (SD).

We can now reconstruct Stalnaker’s argument. Suppose $A$-properties do not strongly supervene on $B^+$-properties. Then there are some possible worlds $w$ and $w'$ and individuals $x$ and $y$ such that $x$ at $w$ is $B^+$-indiscernible but $A$-discernible from $y$ at $w'$. Now, at $w$, $x$ will have some maximal $B$-structural property. Call it $P$. Since $x$ at $w$ is $B^+$-indiscernible from $y$ at $w'$ and $P$ is in $B^+$, at $w'$, $y$ has $P$ as well. But, by

\textsuperscript{36} Strictly speaking, there is no sequence in which \textit{every} object occurs, for the familiar set-theoretical reasons. For current purposes I ignore this presentationally unfortunate fact.

\textsuperscript{37} The properties correspond to the description in the way that everything has the property at some world just in case the corresponding description is true at that world.
the definition of maximal $B$-structural properties, that means that (i) \(x\) has the same $B$-properties at \(w\) that \(y\) has at \(w'\), and (ii) \(w\) and \(w'\) have the same distribution of $B$-maximal properties. By (ii) there are bijections between $D(w)$ and $D(w')$ that are $B$-preserving. By (i), there is one such bijection that maps \(x\) to \(y\). Call it \(f\). Since \(x\) at \(w\) is $A$-discernible from \(y\) at \(w'\), there is at least one $A$-property that \(x\) has at \(w\) and \(f(x) = y\) lacks at \(w'\) or vice versa. Thus, \(f\) is not $A$-preserving. Consequently, if $A$-properties do not strongly supervise on $B'$-properties, $A$-properties do not strongly globally supervise on $B$-properties. 38 Contrapositing: If $A$-properties strongly globally supervise on $B$-properties, $A$-properties strongly supervise on $B'$-properties.

2.3 Philosophical Ramifications

There is considerable debate about the philosophical consequences the various entailments and failures thereof may have. This is especially striking in the case of strong and global supervenience. Kim (1984, 168) initially welcomed the putative truth of the Entailment Claim, since he thought that it attested to the naturalness of both supervenience notions that two paths must lead to the same outcome. Stalnaker (1996, 228) takes his result to show that ‘global supervenience is the appropriate supervenience concept for the statement of many supervenience theses’. Bennett (2004, 501), finally, uses the very same result in conjunction with the weakness of Weak Global Supervenience and Intermediate Global Supervenience to argue for the view that ‘global supervenience [is] little more than a chimera’.

It, thus, seems fair to say that philosophical assessment of the somewhat technical results is not univocal. However, what is clear—if not particularly surprising—is that the more constraints we place on the two sets of properties the more equivalences emerge. Intuitively, what happens is that the properties in the two sets do the work for weaker supervenience notions that is done by the make up of the stronger supervenience notions themselves. The significance of these results

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38 This is the step where proof attempts for Weak Global Supervenience and Intermediate Global Supervenience break down, since only Strong Global Supervenience requires that \(f\) itself is an $A$-isomorphism.
will be relativised by the developments in the next section on explanatory strengthened supervenience notions.

3. Grounding Supervenience

So far we have introduced various supervenience notions and investigated their logical relationships. All of these notions have in common that they can be defined in modal terms. Many philosophers believe that this is as it should be: supervenience is essentially a modal notion, and nothing more. However, there are also arguments available that the so-defined notions are not all a friend of supervenience may have hoped for and should be supplemented. In the first subsection, a case for such a supplementation will be sketched. For some purposes at least, it is worth having stronger notions of supervenience. The second subsection introduces such notions. For reasons of space only forms of individual supervenience will be discussed.

3.1 Supervenience and Priority

As Kim (1990) stresses, many philosophical applications of supervenience rely on supervenience’s being some kind of dependence or (metaphysical) posteriority relation. How else should the fact that $A$-properties supervene on $B$-properties appease someone initially sceptical of $A$-properties, for instance? But, the argument continues, it can be shown that the modal definitions do not define posteriority relations. Thus, the supervenience relations we have met with so far are not adequate for one of the main purposes of supervenience.

One strand of the above argument capitalises on the fact that weak and strong supervenience do not even have the required structural features for being relations of posteriority. Let us start then by listing the structural features of relations at issue. We will follow common practice and say that a relation $R$ has some structural feature within a given

39 Recall the Lewis quotation on p. 96 above. Lewis adds: ‘A supervenience thesis is, in a broad sense, reductionist. But it is a stripped-down form or [sic] reductionism, unencumbered by dubious denials of existence, claims of ontological priority, or claims of translatability’ (Lewis 1983, 358). Cp., e.g., McLaughlin and Bennett 2008, §1, and Horgan 1993, 555.

40 For a particularly clear statement of the argument see Correia 2005, §6.3.
set $A$ of relata, suppressing the specification when $R$ has the structural feature universally.

Slightly idiosyncratically, let us read ‘$xRy$’ as ‘$x$ bears $R$ to $y$’. Then we can define the structural features relevant for our discussion as follows:

**Reflexivity**  
$R$ is reflexive in $A$ $\iff \forall x \in A \ xRx$

**Irreflexivity**  
$R$ is irreflexive in $A$ $\iff \forall x \in A \neg xRx$

**Symmetry**  
$R$ is symmetric in $A$ $\iff \forall x, y \in A \ (xRy \rightarrow yRx)$

**Asymmetry**  
$R$ is asymmetric in $A$ $\iff \forall x, y \in A \ (xRy \rightarrow \neg yRx)$

**Antisymmetry**  
$R$ is antisymmetric in $A$ $\iff \forall x, y \in A \ ((xRy \land yRx) \rightarrow x=y)$

**Transitivity**  
$R$ is transitive in $A$ $\iff \forall x, y, z \in A \ ((xRy \land yRz) \rightarrow xRz)$

**Ordering**  
$R$ is a strict partial ordering of $A$ $\iff$

$R$ is asymmetric and transitive in $A$

Posteriority relations, the converses of priority relations, are strict partial orderings. If something $x$ is posterior to something $y$, then $y$ is not posterior to $x$. And if $x$ is posterior to $y$, which in turn is posterior to $z$, then $x$ is posterior to $z$.

The relation of strong supervenience, on the other hand, is not a partial ordering, and, thus, not a relation of posteriority, whether metaphysical or otherwise. For, strong supervenience is not irreflexive and, thus, not asymmetric either. In fact, it is easy to see that strong supervenience is trivially reflexive in the set of all sets of properties: if $x$ at $w$ is $A$-indiscernible from $y$ at $w'$, $x$ is $A$-indiscernible from $y$ at $w'$. But asymmetry entails irreflexivity. Thus, strong supervenience is not asymmetric. Since strong entails weak supervenience, weak supervenience is not asymmetric either (Kim 1990, 13).

In response to this result, one might try to define close relatives of our old supervenience notions that are partial orderings. Kim (1990, 13) considers two such attempts: (i) build in irreflexivity by definition; (ii) build in asymmetry by definition. That neither attempt fixes the
problem can be seen by considering any set $A$ of modally independent properties $P$ and $Q$ and their negations ($A = \{P, Q, \overline{P}, \overline{Q}\}$) and the corresponding set of possible binary conjunctions of these properties, $B = \{P \cap Q, P \cap \overline{Q}, \overline{P} \cap Q, \overline{P} \cap \overline{Q}\}$.\(^{41}\)

Re (i): $A$ and $B$ are different sets: since $P$ and $Q$ are modally independent, one may have $P$ without having $P \cap Q$ and without having $P \cap \overline{Q}$, one may have $Q$ without having $P \cap Q$ and without having $\overline{P} \cap Q$ etc. But $A$-properties strongly supervene on $B$-properties and $B$-properties strongly supervene on $A$-properties. Thus, strong supervenience between different sets of properties is not asymmetric.

Re (ii): Given transitivity, the relation of non-mutual strong supervenience is guaranteed to be a strict partial ordering. However, the $B$-properties do not non-mutually supervene on $A$-properties in our current example. But the relationship in which the former stand to the latter is one of the paradigm cases of supervenience (in fact, it is the one we used in this survey to illicit a pre-theoretical feel for the relations at issue). Thus, non-mutual strong supervenience excludes too much.

Non-mutual supervenience also excludes too little. It is easy to show that sets that only include properties that are either necessarily exemplified by everything or necessarily exemplified by nothing weakly and strongly supervene on any set of properties. In most cases this supervenience is one-way. For instance, the property of being a number or not being a number non-mutually supervenes on the property of being painted by Leonardo da Vinci.\(^{42}\) But there is no good sense in which the former is metaphysically posterior to the latter.

In short, neither weak nor strong supervenience nor any of their close relatives are posteriority relations. For any philosophical application that requires supervenience to be such a relation, the modally defined notions will not do. The next section sketches how we may

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\(^{41}\) Kim himself uses a different example he borrows from Lombard (1986, 225ff): the sets of perfect sphere volume properties and the set of perfect sphere surface area properties. However, on an intensional individuation of properties the example is problematic, since then we do not have two sets but one.

build on the modal notions to arrive at supervenience candidates that pick out relations of posteriority.

3.2 Supervenience and Grounding

Weak and strong supervenience are not relations of posteriority. There are two consequences one may reasonably draw from this. First, one may take this to show that it is simply mistaken to take supervenience to require posteriority. Second, one may try to improve on extant definitions. Some quotations from authors who have appealed to notions of supervenience without subscribing to the definitions of section 1 above, indicate an attractive way of pursuing the second option.

Thus, Jonathan Bennett writes

I shall say that events are supervenient entities, meaning that all truths about them are logically entailed by and explained or made true by truths that do not involve the event concept. (Bennett 1988, 12)

Hare, discussing the supervenience of the property of being a good picture on natural properties, maintains:

[W]e cannot say ‘[Picture] P is exactly like Q in all respects save this one, that P is a good picture and Q is not’. If we were to say this, we should invite the comment, ‘But how can one be good and the other not, if they are exactly alike? There must be some further difference between them to make one good and the other not.’ (Hare 1952, 81)

Lastly, Sidgwick on moral/non-moral supervenience:

There seems, however, to be this difference between our conceptions of ethical and physical reality: that we commonly refuse to admit in the case of the former—what experience compels us to admit as regards the latter—variations for which we can discover no rational explanation. In the variety of coexistent physical facts we find an arbitrary element in which we have to acquiesce [...]. But within the range of our cognitions

43 See, e.g., McLaughlin and Bennett 2008, §3.5.
of right and wrong, it will generally be agreed that we cannot admit a similar unexplained variation. We cannot judge an action to be right for A and wrong for B, unless we can find in the nature or circumstances of the two some difference which we can regard as reasonable ground for difference in their duties. (Sidgwick 1884, 206)

All three of these authors seem to work with a supervenience notion that is stronger than the purely modally defined notions of weak and strong supervenience. For, Bennett requires truths about other things to explain or make true the truths about events, Hare wants some further property to make a good picture good, and Sidgwick asks for a reasonable ground of the difference between right and wrong actions. Given the tight connection between (non-causal) explanation, making something a certain way (a good picture, true etc.) and the metaphysical notion of ground, the three passages suggest that we should add a grounding requirement to candidate notions of supervenience. This suggestion is taken up in Correia 2005, §6. In what follows I introduce the resulting notions of weak and strong ground-supervenience, and show how they can avoid the charge of not being posteriority relations.

The relevant notion of grounding is further discussed in the article by Kelly Trogdon in this volume, so we may be brief. Let us write ‘φ₁, φ₂,… □ψ’ for ‘φ₁ and φ₂ … together ground ψ’, and ‘[φ]’ for ‘the fact that φ’. We can now take a simplified version of Correia’s alternative characterisation of strong supervenience (recall p. 110 above) and add a grounding requirement: instead of only requiring that for every A-property an individual has there are some B-properties that are strictly sufficient for having the A-property, the new notion of ground-supervenience additionally requires that these B-properties must ground the A-property. That is,

\[ \text{φ₁, φ₂,… □ψ} \]

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44 For the connection between grounding and non-causal explanation see, e.g., the articles collected in Correia and Schnieder 2012. For the connection between making and grounding see Schnieder 2006.
**Strong Ground-Supervenience**

$A$-properties strongly ground-supervene on $B$-properties $\leftrightarrow_{df}$

$\forall x, (P \in A \land x \text{ has } P) \rightarrow \exists QQ (QQ \in B \land x \text{ has } QQ \land \Box \forall y (y \text{ has } QQ \rightarrow [y \text{ has } QQ] \triangleright [y \text{ has } P])).$

We get a corresponding notion of Weak ground-supervenience by deleting the second occurrence of ‘$\Box$’ in the definiens. By way of explaining the workings of Strong Ground-Supervenience reconsider the two sets $A = \{P, Q, \overline{P}, \overline{Q}\}$, and the corresponding set of conjunctions $B = \{P \land Q, P \land \overline{Q}, \overline{P} \land Q, \overline{P} \land \overline{Q}\}$. $A$- and $B$-properties mutually strongly supervene on each other. But $B$-properties strongly ground-supervene on $A$-properties without the latter ground-supervening on the former. For, plausibly, something’s having a property in $B$ is necessarily grounded in that thing’s having some properties in $A$, but not vice versa. Take for instance $P \land Q$, the property of having $P$ and $Q$. Necessarily, everything that has $P$ and has $Q$, has $P \land Q$ as well. In fact, something stronger is true: everything that has $P$ and has $Q$ has $P \land Q$ because it has $P$, and it has $P \land Q$ because it has $Q$. Stronger still, necessarily, whenever something has $P$ and has $Q$, the fact that it has $P$ and the fact that it has $Q$ conspire to ground the fact that it has $P \land Q$. This generalises. Therefore, $B$-properties strongly ground-supervene on $A$-properties. But $A$-properties do not ground-supervene on $B$-properties. For, although, for instance, it is necessary that whatever has $P \land Q$ also has $P$, if something has $P \land Q$ this fact does not ground the fact that it has $P$. The direction of grounding goes from the conjuncts to the conjunction, not the other way around. For this reason, the direction of ground-supervenience goes that way as well.

It can be shown that, generally, strong ground-supervenience is an asymmetric relation that entails strong supervenience. Strong ground-supervenience is, thus, a prime candidate for the job strong supervenience was initially intended to do. This is due to three features of grounding: its factivity and the fact that even partial grounding is asymmetric and transitive. Let us start with factivity. Only facts stand in the grounding relation, and if it is a fact that $p$, then $p$. That is:

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45 These definitions correspond to what Correia (2005, 143) calls strong and weak property supervenience.

46 For more on the logic of ground see Fine forthcoming and Schnieder 2011.
FACTIVITY OF GROUND

□([p_1], [p_2]... ⊨ [q] → (p_1 ∧ p_2 ... ∧ q)).^{47}

Given Factivity of Ground, the definiens of (StrongC) on p. 136 is a straightforward consequence of the definiens of Strong ground-supervenience.

Let us write ‘φ_1 ≥ φ_2’ for ‘φ_1 at least partially grounds φ_2’, defined as follows:

PARTIAL GROUND

φ_1 ≥ φ_2 ↔ def. φ_1 ⊨ φ_2 ∨ ∃ψφ φ_1, ψψ ⊨ φ_2.

Some fact at least partially grounds another if the former grounds the latter either by itself or with the help of some further facts. Now, in the debate about grounding it is generally accepted that partial grounding (and, thus, grounding as well) is asymmetric and transitive:

ASYMMETRY OF PARTIAL GROUND

□∀φ, a □(φ ≥ φ → ¬ψ ≥ φ)

TRANSITIVITY OF PARTIAL GROUND

□∀φ_1,φ_2,φ_3 □((φ_1 ≥ φ_2 ∧ φ_2 ≥ φ_3) → φ_1 ≥ φ_3).

Given these two structural features of grounding, it is quite straightforward to see that strong ground-supervenience must be asymmetric (within the set of sets that include exemplifiable properties).^{48} For, suppose A-properties strongly ground-supervene on B-properties. Then for every A-property P had by some possible individual x there are some B-properties QQ that x has such that, necessarily, if any y has all of QQ, the fact that y has P is grounded in the fact that y has

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^{47} Strictly speaking, the factivity principle should use quantification into sentence position in the following way to ensure that only facts stand in the grounding relation:

□∀x_1, x_2 ... y (x_1, x_2 ... ⊨ y → ∃p_1, p_2 ... q (x_1 = [p_1] ∧ x_2 = [p_2] ... ∧ y = [q] ∧ p_1 ∧ p_2 ... ∧ q)).

Since this is non-standard, I leave it at the schema in the main text.

^{48} The parenthetical remark is necessary to exclude the trivial case. For, take any set that includes only unexemplifiable properties. Such a set trivially strongly ground-supervenes on itself. Arguably, such a case should be excluded by definition.
all of \( QQ \). For each property \( Q \) that is one of \( QQ \) the fact that \( y \) has all of \( QQ \) is at least partially grounded in the fact that \( y \) has \( Q \). Thus, by the *Transitivity of Partial Ground*, for each \( Q \) that is one of \( QQ \), the fact that \( y \) has \( Q \) at least partially grounds the fact that \( y \) has \( P \). By the *Asymmetry of Partial Ground*, finally, \( P \) does not even partially ground \( Q \). Since the choice of \( P \) and \( QQ \) was arbitrary, whenever strong ground-supervenience relates an \( A \)-property to some \( B \)-properties, it does not also relate any of the \( B \)-properties to properties among which is the \( A \)-property. The same holds, *mutatis mutandis* for the corresponding relation of weak ground-supervenience.

Let us sum up. The relations of weak and strong supervenience are not partial orderings and, thus, cannot be relations of metaphysical posteriority. The corresponding relations of ground-supervenience entail them and can be shown to be asymmetric (excluding the trivial case). In virtue of this fact, they have a good claim of capturing interesting supervenience claims. If this is so, two of the main protagonists of this collection—supervenience and grounding—are not in competition. To the contrary, grounding may ground supervenience.\(^{49}\)

**References**


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